

# Relation between $T_{cc,bb}$ and $X_{c,b}$ from QCD

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## Abstract

We have studied, using double ratio of QCD (spectral) sum rules, the ratio between the masses of  $T_{cc}$  and  $X(3872)$  assuming that they are respectively described by the  $D - D^*$  and  $D - \bar{D}^*$  molecular currents. We found (within our approximation) that the masses of these two states are almost degenerate. Since the pion exchange interaction between these mesons is exactly the same, we conclude that if the observed  $X(3872)$  meson is a  $DD^* + c.c.$  molecule, then the  $DD^*$  molecule should also exist with approximately the same mass. An extension of the analysis to the  $b$ -quark case leads to the same conclusion. We also study the SU(3) breakings for the  $T_{QQ}^s/T_{QQ}$  mass ratios. Motivated by the recent Belle observation of two  $Z_b$  states, we revise our determination of  $X_b$  by combining results from exponential and FESR sum rules.

**Keywords:** QCD spectral sum rules, non-perturbative methods, exotic multi-quark states, heavy quarkonia.

## 1. Introduction

The existence of exotic hadrons is a long-standing problem. By exotic we mean a state whose quantum numbers and main properties cannot be explained by a simple quark-antiquark or three-quark configuration.

The  $X(3872)$  resonance (assumed to be an  $1^{++}$  axial vector meson) has, indeed, stimulated *many activities in the physics of hadrons*. It was discovered by BELLE in  $B$ -decays [1], and confirmed by BABAR [2], CDF [3] and D0 [4]. It is rather narrow, with a width  $\leq 2.3$  MeV. Its most popular picture which consists of a molecular configuration,  $DD^* + \bar{D}D^*$ , with  $J^{PC} = 1^{++}$ , has been attributed to the narrow ( $\leq 2.3$  MeV width)  $X(3872)$  <sup>1</sup>.

The case of the four-quark state ( $QQ\bar{u}\bar{d}$ ) with quantum numbers  $I = 0$ ,  $J = 1$  and  $P = +1$  which, following ref.[6], we call  $T_{QQ}$ , is especially interesting. As already noted previously [6, 7], the  $T_{bb}$  or  $T_{cc}$  states with  $J^P = 1^+$  cannot split into a pair of two  $\bar{B}$  or two  $D$  mesons which is restricted to  $J^P = 0^+, 2^+, \dots$ . If their masses are below the  $\bar{B}\bar{B}^*$  or  $DD\pi$  thresholds, these decays are also forbidden. As a result,  $T_{QQ}$  becomes stable with respect to strong interaction, and must decay radiatively, or even weakly if the mass becomes lower than the threshold made of two pseudoscalar mesons.

## 2. $T_{QQ}$ from potential models

In constituent models with a flavor-independent central potential, the stability of  $(QQ\bar{q}\bar{q})$  configurations comes from a favorable effect when the charge-conjugation symmetry is broken, as noted many years ago [8]. This is the same mechanism by which, in QED, the loosely bound positronium molecule evolves into the very stable hydrogen molecule.

It is worth noting that in the large  $m_Q$  limit, the light degrees of freedom cannot resolve the closely bound  $QQ$  system. This results in bound states similar to the  $\bar{\Lambda}_Q$  states, with  $QQ$  playing the role of the heavy antiquark [9].

The  $(QQ\bar{q}\bar{q})$  states have been studied using a variety of simple or elaborated potential models [7, 8, 10–12]. The corresponding four-body problem is very delicate. For instance, an expansion on harmonic-oscillator states was used in [11]. It is efficient for deep binding but converges very slowly for weak binding. If truncated, this expansion may fail to demonstrate stability with potentials that do bind, because it lacks explicit  $(Q\bar{q}) - (Q\bar{q})$  components, which are important near threshold [7], and are included in the Gaussian expansion sketched in [12] and systematically developed in [6]. See, also, Ref. [13] for a discussion about the four-quark problem. All authors agree that such states become bound when the quark over the antiquark mass ratio becomes sufficiently large. Detailed four-body calculations, using a pairwise central potential supplemented by a chromomagnetic interaction, indicate that  $T_{bb}$  is rather well bound, and  $T_{cc}$  possibly bound by a few MeV below  $DD^*$ . For instance, the prediction of Ref. [6] is, in units of MeV:

$$M_{T_{cc}} = 3876 \sim 3905, \quad M_{T_{bb}} = 10519 \sim 10651. \quad (1)$$

A non-pairwise confinement has also been considered [14], inspired by the large coupling regime of QCD, where it is shown

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<sup>1</sup>For references about some other possible interpretations of the  $X(3872)$  see, e.g., [5].

[15] that it is more favorable to build stable tetraquarks. In this improved quark model, as well as in conventional quark models, it is found that  $(QQ\bar{q}\bar{q})$  has an energy lower than  $(Q\bar{Q}q\bar{q})$ .

Another variant was considered in [16], with a chiral potential model, which includes meson-exchange forces between quarks, instead of the chromomagnetic interaction.

The existence of a  $D\bar{D}^* + \bar{D}D^*$  molecule was predicted in ref. [17] on the basis of the pion-exchange dynamics<sup>2</sup>. Here, the pion is exchanged between the hadrons, as in the Yukawa theory of nuclear forces. The  $DD^*\pi$  and  $\bar{D}\bar{D}^*\pi$  vertices are identical, as well as the  $D^*D^*\pi$  and  $\bar{D}^*\bar{D}^*\pi$  ones. There is only an overall change of sign, due to the  $G$ -parity of the pion. Therefore, if the pion-exchange dynamics<sup>3</sup> is able to bind the  $D\bar{D}^* + \bar{D}D^*$  molecule, the same is true for the  $DD^*$  molecule. The difference between these two states can only come from the short-range part of the interaction.

### 3. $T_{QQ}$ from QCD (spectral) sum rules

The first study of tetraquarks with two heavy quarks within QCD (spectral) sum rules (QCDSR) was done in [19] by using diquark-antidiquark current. This study is revisited and improved in the present paper. Our aim is also to compare in detail the  $(QQ\bar{q}\bar{q})$  and  $(Q\bar{Q}q\bar{q})$  configurations. Such a comparison is attempted in ref. [20], where the authors study heavy tetraquarks using a crude color-magnetic interaction, with flavor symmetry breaking corrections. They assume that the Belle resonance,  $X(3872)$ , is a  $c\bar{q}\bar{c}q$  tetraquark, and use its mass as input to determine the mass of other tetraquark states. They obtain, in units of MeV:

$$M_{T_{cc}} \simeq 3966, \quad M_{T_{bb}} \simeq 10372, \quad (2)$$

in agreement with the previous results in Eq. (1) and the ones from QCD (spectral) sum rule, in units of GeV [19]:

$$M_{T_{cc}} = 4.2 \pm 0.2, \quad M_{T_{bb}} = 10.2 \pm 0.3. \quad (3)$$

The short-range part of the interaction can be tested by the QCD (spectral) sum rules approach [21–23]. Therefore, in this work, we study the ratio of the masses of the  $T_{cc}$  and  $X(3872)$  states, by using the double ratios of sum rules (DRSR) introduced in [24], which is widely applied for accurate determinations of the ratios of couplings and masses [25–31] and form factors [32]. This accuracy is reached due to partial cancellations of the systematics of the method and of the QCD corrections in the DRSR. More recently, the DRSR was used to study different possible currents for the  $X(3872)$  [27]. It was found that (within the accuracy of the method) the different structures ( $\bar{3} - 3$  and  $\bar{6} - 6$  tetraquarks and  $D\bar{D}^* + \bar{D}D^*$  molecule) lead to the same prediction for the mass. This result could indicate that the short-range part of the interaction alone may not be sufficient to reveal the nature of the  $X(3872)$ .

<sup>2</sup> For further references on this approach, see e.g. [18].

<sup>3</sup> Usually, the  $G$  parity rule transforms an attractive potential into a repulsive one. Here, however, it only changes the sign of the transition potential  $D\bar{D}^* \rightarrow D^*\bar{D}$ , and thus just a phase in the two-component bound state wave function.

#### 3.1. Two-point functions and forms of the sum rules

The two-point functions of the  $X(3872)$  (assumed to be an  $1^{++}$  axial vector meson) and the  $T_{cc}$  (assumed to be a  $J^P = 1^+$  state) is defined as:

$$\begin{aligned} \Pi_i^{\mu\nu}(q) &\equiv i \int d^4x e^{iq \cdot x} \langle 0 | T [j_i^\mu(x) j_i^{\nu\dagger}(0)] | 0 \rangle \\ &= -\Pi_{1i}(q^2)(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \Pi_{0i}(q^2) \frac{q^\mu q^\nu}{q^2}, \end{aligned} \quad (4)$$

where  $i = X, T_{cc}$ . The two invariants,  $\Pi_1$  and  $\Pi_0$ , appearing in Eq. (4) are independent and have respectively the quantum numbers of the spin 1 and 0 mesons.

We assume that the  $X(3872)$  and  $T_{cc}$  states are described by the molecular currents:

$$\begin{aligned} j_X^\mu(x) &= \left(\frac{g}{\Lambda}\right)_{\text{eff}}^2 \frac{1}{\sqrt{2}} \left[ (\bar{q}_a(x) \gamma_5 c_a(x) \bar{c}_b(x) \gamma^\mu q_b(x)) \right. \\ &\quad \left. - (\bar{q}_a(x) \gamma^\mu c_a(x) \bar{c}_b(x) \gamma_5 q_b(x)) \right]. \end{aligned} \quad (5)$$

and

$$j_{T_{cc}}^\mu(x) = \left(\frac{g'}{\Lambda}\right)_{\text{eff}}^2 \left( \bar{q}_a(x) \gamma_5 c_a(x) \bar{q}_b(x) \gamma^\mu c_b(x) \right), \quad (6)$$

where  $a$  and  $b$  are color indices.

In the molecule assignment, it is assumed that there is an effective local current and the meson pairs are weakly bound by a van der Waals force in a Fermi-like theory with a strength  $(g/\Lambda)_{\text{eff}}^2$  which has nothing to do with the quarks and gluons inside each meson.

Due to its analyticity, the correlation function,  $\Pi_{1i}$  in Eq. (4), can be written in terms of a dispersion relation:

$$\Pi_{1i}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho_i(s)}{s - q^2} + \dots, \quad (7)$$

where  $\rho_i(s) \equiv \text{Im}[\Pi_{1i}(s)]$  is the spectral function.

The sum rule is obtained by evaluating the correlation function in Eq. (4) in two ways: using the operator product expansion (OPE) and using the information from hadronic phenomenology. In the OPE side we work at leading order of perturbation theory in  $\alpha_s$ , and we consider the contributions from condensates up to dimension six. In the phenomenological side, the correlation function is estimated by inserting intermediate states for the  $X$  and  $T_{cc}$  states via their couplings  $\lambda_i$  to the molecular currents:

$$\langle 0 | j_i^\mu | M_i \rangle = \lambda_i \epsilon^\mu. \quad (8)$$

where  $M_i \equiv X, T_{cc}$ ,  $j_i^\mu$  are the currents in Eqs. (5) and (6).

Using the ansatz: “one resonance”  $\oplus$  “QCD continuum”, where the QCD continuum comes from the discontinuity of the QCD diagrams from a continuum threshold  $t_c$ , the phenomenological side of Eq. (4) can be written as:

$$\Pi_{\mu\nu}^{\text{phen}}(q^2) = \frac{\lambda_i^2}{M_i^2 - q^2} \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{M_i^2} \right) + \dots, \quad (9)$$

where the Lorentz structure  $g_{\mu\nu}$  projects out the  $1^+$  state. The dots denote higher axial-vector resonance contributions that

will be parametrized, as usual, by the QCD continuum. After making an inverse-Laplace (or Borel) transform on both sides, and transferring the continuum contribution to the QCD side, the moment sum rule and its ratio read:

$$\begin{aligned}\mathcal{F}_i(\tau) &\equiv \lambda_i^2 e^{-M_i^2 \tau} = \int_{4m_c^2}^{\tau_c} ds e^{-s\tau} \rho_i(s) \\ \mathcal{R}_i(\tau) &\equiv -\frac{d}{d\tau} \log \mathcal{F}_i(\tau) \simeq M_i^2\end{aligned}\quad (10)$$

where  $\tau \equiv 1/M^2$  is the sum rule variable with  $M$  being the inverse-Laplace (or Borel) mass. In the following, we shall work with the DRSR [24]:

$$r_{T_{cc}/X} \equiv \sqrt{\frac{\mathcal{R}_{T_{cc}}}{\mathcal{R}_X}} \simeq \frac{M_{T_{cc}}}{M_X}. \quad (11)$$

### 3.2. QCD expression of the spectral functions

The QCD expressions of the spectral densities of the two-point correlator associated to the current in Eq. (5) are given in ref. [33]. Up to dimension-six condensates the expressions associated to the current in Eq. (6), in the structure  $g_{\mu\nu}$  are:

$$\begin{aligned}\rho(s) &= \rho^{pert}(s) + \rho^{\langle \bar{q}q \rangle}(s) + \\ &\quad \rho^{\langle G^2 \rangle}(s) + \rho^{mix}(s) + \rho^{\langle \bar{q}q \rangle^2}(s),\end{aligned}\quad (12)$$

with

$$\begin{aligned}\rho^{pert}(s) &= \frac{1}{3.2^{13}\pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} (1-\alpha-\beta) \times \\ &\quad \times [(\alpha+\beta)m_c^2 - \alpha\beta s]^3 \left[ 2m_c^2(13\alpha^2 + \right. \\ &\quad \left. + 13\alpha\beta + 7\alpha + 5) - 15\alpha\beta s(1+\alpha+\beta) \right], \\ \rho^{\langle \bar{q}q \rangle}(s) &= -\frac{m_c \langle \bar{q}q \rangle}{2^6 \pi^4} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} (1+\alpha+\beta) \times \\ &\quad \times [(\alpha+\beta)m_c^2 - \alpha\beta s]^2, \\ \rho^{\langle G^2 \rangle}(s) &= \frac{\langle g^2 G^2 \rangle}{2^{13} 3.2^6 \pi^6} \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha^2} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left\{ 12\alpha\beta(5\alpha + \right. \\ &\quad \left. + 5\beta - 3) [(\alpha+\beta)m_c^2 - \alpha\beta s]^2 + m_c^4(1-\alpha-\beta)^2 \times \right. \\ &\quad \times \alpha^2(5+\alpha+\beta) + m_c^2(1-\alpha-\beta) \left[ 9\alpha^2(2 + \right. \\ &\quad \left. + 3\alpha + 4\beta) + \alpha\beta(2+2\alpha+11\beta) + \right. \\ &\quad \left. + 15\alpha - 2\beta \right] [(\alpha+\beta)m_c^2 - \alpha\beta s] \Big\}, \\ \rho^{mix}(s) &= -\frac{m_c \langle \bar{q}g\sigma.Gq \rangle}{3.2^8 \pi^4} \left\{ 7 \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha(1-\alpha)} [m_c^2 + \right. \\ &\quad \left. - \alpha(1-\alpha)s] - \int_{\alpha_{min}}^{\alpha_{max}} \frac{d\alpha}{\alpha} \int_{\beta_{min}}^{1-\alpha} \frac{d\beta}{\beta^2} [12\alpha^2 + \right.\end{aligned}$$

$$\begin{aligned}&\left. + 17\alpha\beta - \beta \right] [(\alpha+\beta)m_c^2 - \alpha\beta s] \Big\}, \\ \rho^{\langle \bar{q}q \rangle^2}(s) &= \frac{\rho \langle \bar{q}q \rangle^2}{3.2^6 \pi^2} \int_{\alpha_{min}}^{\alpha_{max}} d\alpha [13m_c^2 - 5\alpha(1-\alpha)s],\end{aligned}\quad (13)$$

where:  $m_c$ ,  $\langle g^2 G^2 \rangle$ ,  $\langle \bar{q}q \rangle$ ,  $\langle \bar{q}g\sigma.Gq \rangle$  are respectively the charm quark mass, gluon condensate, light quark and mixed condensates;  $\rho$  indicates the violation of the four-quark vacuum saturation. The integration limits are given by:

$$\begin{aligned}\alpha_{min} &= \frac{1}{2}(1-v), \quad \alpha_{max} = \frac{1}{2}(1+v) \\ \beta_{min} &= \alpha m_c^2 / (s\alpha - m_c^2)\end{aligned}\quad (14)$$

where  $v$  is the  $c$ -quark velocity:

$$v \equiv \sqrt{1 - 4m_c^2/s}. \quad (15)$$

### 3.3. $T_{cc}/X$ ratio of masses

In the following, we shall extract the mass ratio  $T_{cc}/X$  using the DRSR in Eq. (11). For the numerical analysis we shall introduce the renormalization group invariant quantities  $\hat{m}_s$  and  $\hat{\mu}_q$  [34, 35]:

$$\begin{aligned}\bar{m}_s(\tau) &= \frac{\hat{m}_s}{(-\log \sqrt{\tau}\Lambda)^{-2/\beta_1}} \\ \langle \bar{q}q \rangle(\tau) &= -\hat{\mu}_q^3 (-\log \sqrt{\tau}\Lambda)^{-2/\beta_1} \\ \langle \bar{q}g\sigma.Gq \rangle(\tau) &= -m_0^2 \hat{\mu}_q^3 (-\log \sqrt{\tau}\Lambda)^{-1/3\beta_1},\end{aligned}\quad (16)$$

where  $\beta_1 = -(1/2)(11 - 2n/3)$  is the first coefficient of the  $\beta$  function for  $n$  flavours. We have used the quark mass and condensate anomalous dimensions reported in [23]. We shall use the QCD parameters in Table 1. At the scale where we shall work, and using the parameters in Table 1, we deduce:  $\rho = 2.1 \pm 0.2$ , which controls the deviation from the factorization of the four-quark condensates. We shall not include the  $1/q^2$  term discussed in [36, 37], which is consistent with the LO approximation used here as the latter has been motivated by a phenomenological parametrization of the larger order terms of the QCD series.

Using QCD (spectral) sum rules, one can usually estimate the mass of the  $X$ -meson, from the ratio  $\mathcal{R}_X$  in Eq. (10), which is related to the spectral densities obtained from the current (5). A tetraquark current for the  $X(3872)$  was used in ref. [26]. At the sum rule stability point and using a slightly different (though consistent) set of QCD parameters than in Table 1, one obtains, with a good accuracy, for  $m_c = 1.26$  GeV [26] <sup>4</sup>:

$$M_X \simeq \sqrt{\mathcal{R}_X} = (3925 \pm 127) \text{ MeV}, \quad (17)$$

and the correlated continuum threshold value fixed simultaneously by the Laplace and finite energy sum rules (FESR) sum rules:

$$\sqrt{t_c} \simeq (4.15 \pm 0.03) \text{ GeV}. \quad (18)$$

<sup>4</sup>The use of  $m_c = 1.47$  GeV increases the central value by about (160 ~ 200) MeV.

Table 1: QCD input parameters. For the heavy quark masses, we use the range spanned by the running  $\overline{MS}$  mass  $\overline{m}_Q(M_Q)$  and the on-shell mass from QCD (spectral) sum rules compiled in page 602,603 of the book in [23]. The values of  $\Lambda$  and  $\hat{\mu}_q$  have been obtained from  $\alpha_s(M_\tau) = 0.325(8)$  [38] and from the running masses:  $(\overline{m}_u + \overline{m}_d)(2) = 7.9(3)$  MeV [39]. The original errors have been multiplied by 2 for a conservative estimate of the errors.

Parameters	Values	Ref.
$\Lambda(n_f = 4)$	$(324 \pm 15)$ MeV	[38, 40]
$\hat{\mu}_q$	$(263 \pm 7)$ MeV	[23, 39]
$\hat{m}_s$	$(0.114 \pm 0.021)$ GeV	[23, 39, 40]
$m_c$	$(1.23 \sim 1.47)$ GeV	[23, 39–43]
$m_b$	$(4.2 \sim 4.7)$ GeV	[23, 39–42]
$m_0^2$	$(0.8 \pm 0.2)$ GeV <sup>2</sup>	[28, 44, 45]
$\langle \alpha_s G^2 \rangle$	$(6 \pm 2) \times 10^{-2}$ GeV <sup>4</sup>	[38, 41, 46–49, 51–53]
$\rho \alpha_s \langle \bar{d}d \rangle^2$	$(4.5 \pm 0.3) \times 10^{-4}$ GeV <sup>6</sup>	[38, 44, 46]

In ref. [27] it was obtained that the DRSR for the tetraquark current and for the molecular current is:

$$r_{mol/3} = \sqrt{\frac{\mathcal{R}_{mol}}{\mathcal{R}_3}} \simeq 1.00, \quad (19)$$

with a negligible error. Therefore, the result in Eq. (18) is the same for the current in Eq. (5). Although the uncertainty in Eq. (17) is still large, considering the fact that this result was obtained in a Borel region where there is pole dominance and OPE convergence, one can say that the QCD sum rules supports the existence of such a state and that the value obtained for  $M_X$  is in reasonable agreement with the experimental candidate [40]:

$$M_X|_{exp} = (3872.2 \pm 0.8) \text{ MeV}. \quad (20)$$

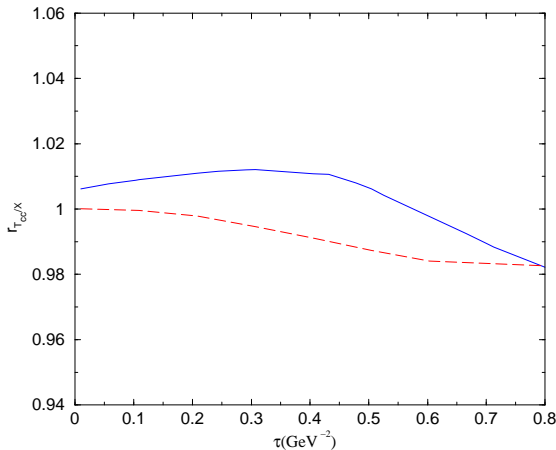


Figure 1: The double ratio  $r_{T_{cc}/X}$  defined in Eq. (11) as a function of  $\tau$  for  $\sqrt{t_c} = 4.15$  GeV and for two values of  $m_c = 1.23$  (solid line) and  $1.47$  GeV (dashed line).

We now study the DRSR of the  $T_{cc}/X$  defined in Eq. (11). In Fig. 1, we show the  $\tau$ -dependence of the ratio for  $\sqrt{t_c} = 4.15$  GeV and for two values of  $m_c = 1.23$  GeV and  $1.47$  GeV.

From Fig. 1 one can see that there is a  $\tau$ -stability around  $\tau \simeq 0.4 \text{ GeV}^{-2}$  and for this value of  $\tau$ , we get:

$$r_{T_{cc}/X} = 1.00 \pm 0.01. \quad (21)$$

In Fig. 2, we show the  $t_c$ -dependence of the ratio for  $\tau = 0.4 \text{ GeV}^{-2}$  and for two values of  $m_c = 1.23$  GeV and  $1.47$  GeV. From this figure one can see that the ratio increases with  $t_c$ . However, considering the large range of  $t_c$  presented in the figure, the ratio does not differ more than 3% from 1.

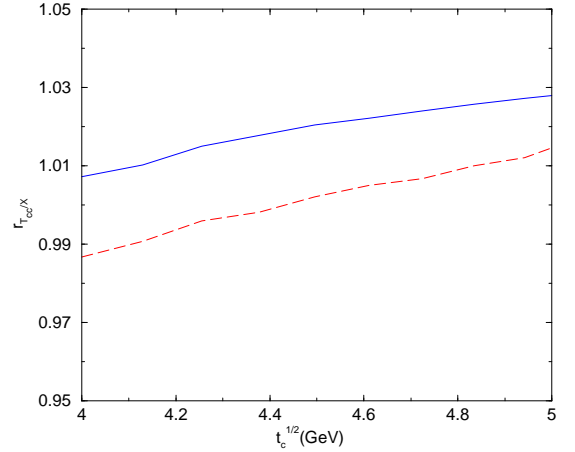


Figure 2: The double ratio  $r_{T_{cc}/X}$  as a function of  $t_c$  for  $\tau = 0.4 \text{ GeV}^{-2}$  and for two values of  $m_c = 1.23$  (solid line) and  $1.47$  GeV (dashed line).

Our analysis has shown that the  $D\bar{D}^* + c.c.$  and  $DD^*$  currents lead to the same mass predictions within the accuracy of the approach. The accuracy of the DRSR is bigger than the normal QCDSR because the DRSR are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions. As mentioned before, this accuracy is reached due to partial cancellations of the systematics of the method and of the QCD corrections in the DRSR. Therefore, if the observed  $X(3872)$  is a molecular  $D\bar{D}^* + c.c.$  state its molecular cousin  $DD^*$  should also be a bound state. Its mass can be obtained by using the experimental mass for the  $X(3872)$  in Eq. (21):

$$M_{T_{cc}} = (3872.2 \pm 39.5) \text{ MeV} \quad (22)$$

### 3.4. $T_{bb}/X_b$ ratio of masses

Using the same interpolating field in Eqs. (5) and (6) with the charm quark replaced by the bottom one, we can analyse the DRSR:

$$r_{T_{bb}/X_b} \equiv \sqrt{\frac{\mathcal{R}_{T_{bb}}}{\mathcal{R}_{X_b}}} \simeq \frac{M_{T_{bb}}}{M_{X_b}}. \quad (23)$$

In Fig. 3, we show the  $\tau$ -dependence of the ratio in Eq. (23) for  $\sqrt{t_c} = 10.5$  GeV and for two values of  $m_b$ . From this figure one can see the ratio is very stable. The same happens for the dependence of this ratio with  $t_c$ , as can be seen by Fig. 4. We get:

$$r_{T_{bb}/X_b} = 1.00. \quad (24)$$

Therefore, we can predict the degeneracy between the masses of the  $T_{bb}$  and of the  $X_b$  given in Eq. (25).

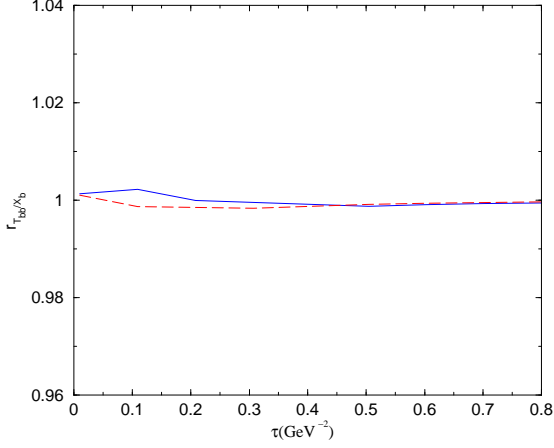


Figure 3: Same as Fig. 1 for  $r_{T_{bb}/X_b}$  for  $\sqrt{t_c} = 10.5$  GeV and for two values of  $m_b = 4.2$  (solid line) and  $4.7$  GeV (dashed line).

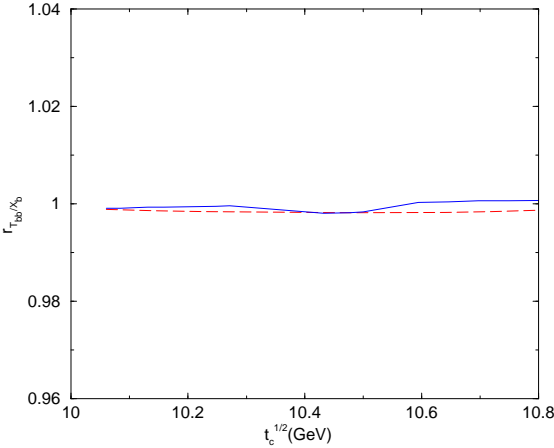


Figure 4: Same as Fig. 2 for  $r_{T_{bb}/X_b}$  for  $\tau = 0.2$   $\text{GeV}^{-2}$  and for two values of  $m_b = 4.2$  (solid line) and  $4.7$  GeV (dashed line).

#### 4. Revisiting the determination of the $X_b$ mass

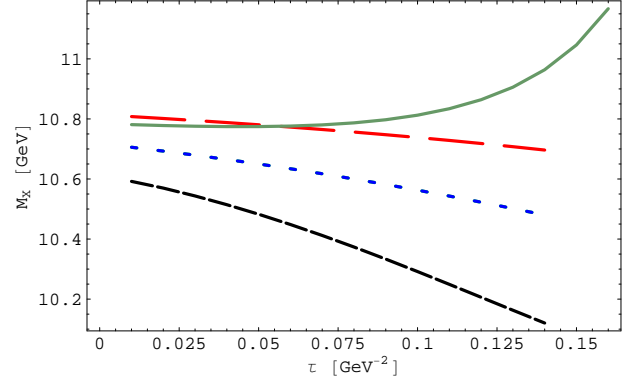


Figure 5:  $M_{X_b}$  in GeV as a function of  $\tau$  in  $\text{GeV}^{-2}$  from Laplace sum rule for  $m_b = 4.7$  GeV and  $\sqrt{t_c} = 11$  GeV: long dashed (red) : (1)= perturbative (Pert) contribution; small dashed (blue): (2)= Pert+ $\langle\bar{d}d\rangle + \langle\alpha_s G^2\rangle$  contributions (the one of the gluon condensate is relatively negligible); continuous (olive): (3)=(2)+ mixed condensate  $\langle g\bar{d}Gd\rangle$ ; medium dashed (black): (4)=(3)+ $\langle\bar{d}d\rangle^2$ .

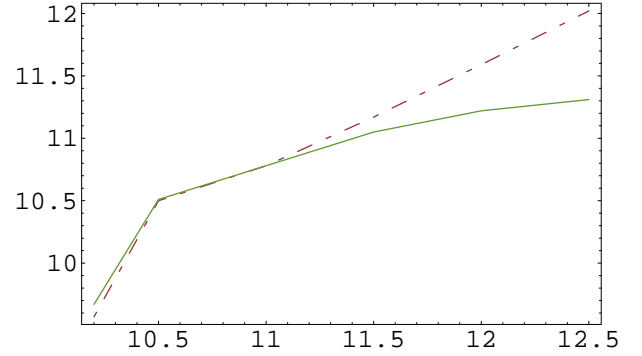


Figure 6:  $M_{X_b}$  in GeV as a function of  $\sqrt{t_c}$  in GeV from FESR (dashed curve) and the value at the  $\tau$ -stability from Laplace sum rule for  $m_b = 4.7$  GeV. The OPE has been truncated at  $D = 5$ .

The  $X_b$  was studied in ref. [26]. At the sum rule stability point and using the perturbative  $\overline{MS}$ -mass  $m_b(m_b) = 4.24$  GeV, they get:

$$10.06 \text{ GeV} \leq M_{X_b} \leq 10.50 \text{ GeV} , \quad (25)$$

for  $10.2 \text{ GeV} \leq \sqrt{t_c} \leq 10.8 \text{ GeV}$ , while combining the Laplace sum rule with FESR, they obtain a slightly lower but more precise value :

$$M_{X_b} = (10.14 \pm 0.10) \text{ GeV} . \quad (26)$$

We complete the previous analysis by using here the value of the on-shell mass  $m_b = 4.7$  GeV due to the ambiguous definition of the quark mass used as we work to leading order of radiative corrections. For a close comparison with the analysis in [26], we shall work with the two-point function associated to the four-quark current <sup>5</sup>. We notice that the contribution of

<sup>5</sup>The result using the  $D\bar{D}^*$  molecule current would be the same as we have shown in [27] that the masses obtained from the four-quark and molecule currents are degenerate.

the  $D = 6$  condensate  $\langle \bar{q}q \rangle^2$  destabilizes the result [medium dashed (black) curve]<sup>6</sup>, which is restored if one adds the  $D = 8$  condensate given in [26]. However, we refrain to add such a term due to the eventual uncertainties for controlling the high-dimension condensate contributions (violation of factorization, complete  $D = 8$  contributions) and find safer to limit the analysis to the  $D = 5$  contribution like in Ref. [26]. In this way, the ratio of sum rules present  $\tau$ -stability at about  $0.1 \text{ GeV}^{-2}$  (continuous curve in Fig 5). We show in Fig. 6 the  $t_c$ -behaviour of  $M_{X_b}$  versus the continuum threshold  $t_c$ , where a common solution is obtained in units of GeV:

$$M_{X_b} = 10.50 \sim 10.78 \quad \text{for} \quad \sqrt{t_c} = 10.5 \sim 11.0, \quad (27)$$

which combined with the result in Eq. (26) leads to the conservative range of values:

$$M_{X_b} = 10.14 \sim 10.78 \quad \text{for} \quad \sqrt{t_c} \approx M_{X'_b} = 10.5 \sim 11.0, \quad (28)$$

where one can notice the relatively small mass-difference between  $\sqrt{t_c}$  and  $M_{X_b}$  eventually signaling the nearby location of the radial excitations as mentioned in [26].

Very recently the Belle Collaboration studied the  $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^\pm$  and  $\Upsilon(5S) \rightarrow h_b(mP)\pi^\pm$  ( $n = 1, 2, 3$  and  $m = 1, 2$ ) decay processes looking for resonant substructures. They found two narrow states in units of MeV:

$$M_{Z_b} = 10610 \quad \text{and} \quad M_{Z'_b} = 10650, \quad (29)$$

with a hadronic width in units of MeV:

$$\Gamma_{Z_b} = 15.6 \pm 2.5 \quad \text{and} \quad \Gamma_{Z'_b} = 14.4 \pm 3.2, \quad (30)$$

respectively [54]. The analysis of the  $Z_b$  states decay in the channel  $Z_b^+ \rightarrow \Upsilon(2S)\pi^+$  favors the  $J^P = 1^+$  assignment, which is the same as the one of the  $X_b$ , although  $X_b$  has positive charge conjugation and the neutral partner of  $Z_b$  should have negative charge conjugation.

Considering the errors in Eq. (28) and the small mass difference between  $X_b$  and  $\sqrt{t_c}$ , it is difficult to identify the two observed  $Z_b$  states with the  $X_b$ .

## 5. $SU(3)$ Mass-splittings

We extend the previous analysis to study the ratio between the strange  $T_{cc}^s$  ( $(cc\bar{s}\bar{s})$ ) and non-strange  $T_{cc}$  states. The QCD expression for the spectral function proportional to  $m_s$  for the current in Eq. (6) is:

$$\rho^{m_s}(s) = \frac{m_s}{28\pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \left\{ \beta \frac{7[m_c^2 - \alpha(1-\alpha)s]^2}{(1-\alpha)} + \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} [(\alpha + \beta)m_c^2 - \alpha\beta s] \left[ \beta \left( m_c^2(21 + \alpha + \beta) \right. \right. \right.$$

<sup>6</sup>We have neglected the contribution of the triple gluon condensate  $\langle g^3 f_{abc} G^3 \rangle$  due to the  $(1/16\pi^2)^2$  loop factor suppression compared to  $\langle \bar{q}q \rangle^2$ .

$$\left. -6 \left[ (\alpha + \beta)m_c^2 - \alpha\beta s \right] \right) - \frac{m_c}{2\pi^2\alpha\beta^2} (3 + \alpha + \beta) \times (1 - \alpha - \beta) \left[ (\alpha + \beta)m_c^2 - \alpha\beta s \right]^2 \left. \right\}. \quad (31)$$

We start by studying the DRSR:

$$r_{T_{cc}^s/T_{cc}}^s \equiv \sqrt{\frac{\mathcal{R}_{T_{cc}^s}^s}{\mathcal{R}_{T_{cc}}^s}} \simeq \frac{M_{T_{cc}^s}}{M_{T_{cc}}}. \quad (32)$$

We study the convergence of the DRSR versus  $\tau$  in Fig 7, where

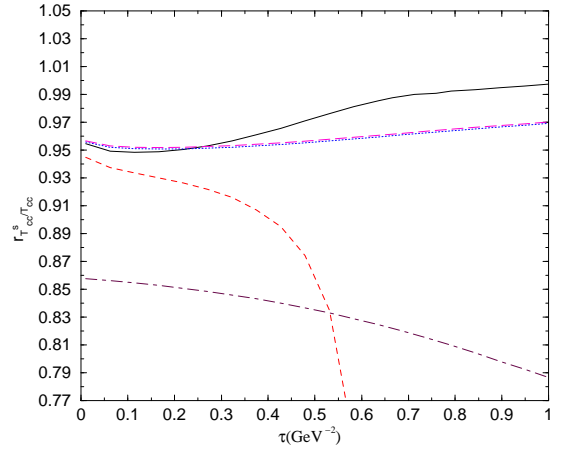


Figure 7: Different QCD contributions to the DRSR  $r_{T_{cc}^s/T_{cc}}^s$  defined in Eq. (32) for  $\sqrt{t_c} = 4.15 \text{ GeV}$  and  $m_c = 1.23 \text{ GeV}$ : (1)=Pert+ $m_s$ : dot-dashed (maroon); (2)=(1) +  $\langle \bar{q}q \rangle$ : long-dashed (magenta); (3)=(2) +  $\langle \alpha_s G^2 \rangle$ : dotted (blue); (4)=(3) +  $\langle g \bar{q} G q \rangle$ : dashed (red); (5)=(4) +  $\langle \bar{q}q \rangle^2$ : continuous (black).

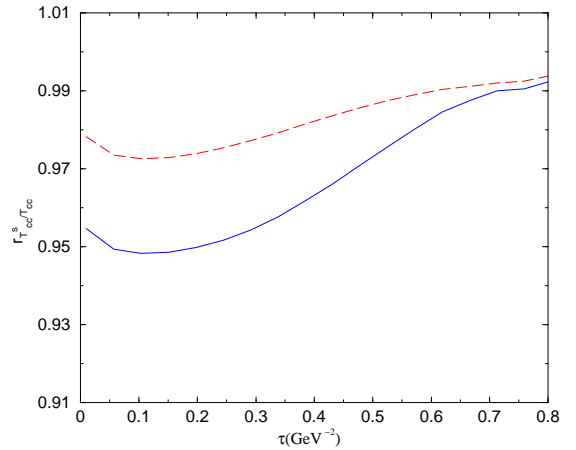


Figure 8: Same as Fig. 1 for  $r_{T_{cc}^s/T_{cc}}^s$  defined in Eq. (32) for  $\sqrt{t_c} = 4.15 \text{ GeV}$  and for two values of  $m_c = 1.23$  (solid line) and  $1.47 \text{ GeV}$  (dashed line).

the show the strength of the different contributions in the OPE. We note that we have  $\tau$ -stability around  $\tau = 0.2 \text{ GeV}^{-2}$ , while the OPE breaks down for  $\tau \geq 0.5 \text{ GeV}^{-2}$ .

In Fig. 8, we show the  $\tau$ -dependence of the ratio in Eq. (32), for  $\sqrt{t_c} = 4.15$  GeV and for two values of  $m_c$ . From this Figure one can deduce around the  $\tau$ -stability:

$$r_{T_{cc}^s/T_{cc}} = 0.95 \sim 0.98, \quad (33)$$

which gives a smaller mass for  $T_{cc}^s$  than for  $T_{cc}$ . This result is similar to the result obtained for  $X^s$  in ref. [26]. However, in this case, the decrease in the mass is even bigger than the obtained for  $X^s$ :  $r_{X^s/X} = 0.984 \pm 0.009$ . This result is consistent with what is obtained from the DRSR:

$$r_{T_{cc}^s/X^s} \equiv \sqrt{\frac{\mathcal{R}_{T_{cc}^s}}{\mathcal{R}_{X^s}}} \simeq \frac{M_{T_{cc}^s}}{M_{X^s}}, \quad (34)$$

as can be seen by Fig. 9.

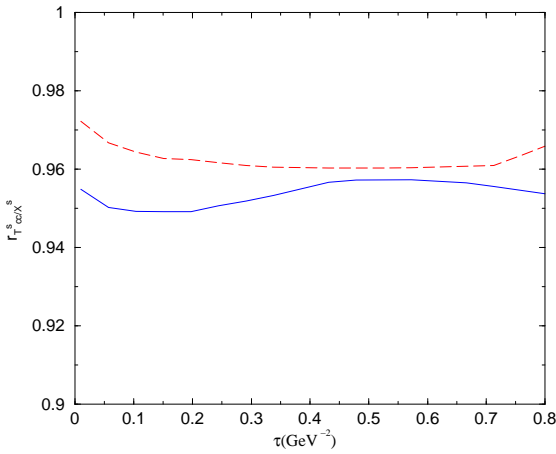


Figure 9: Same as Fig. 1 for  $r_{T_{cc}^s/X^s}$ . The solid and dashed lines are for  $m_c = 1.23$  and  $1.47$  GeV respectively.

In Fig. 9 we have used  $t_c = 4.15$  GeV. However, the result is very stable as a function of  $t_c$ , as can be seen in Fig. 10.

Since for the  $T_{cc}^s$  state, considered as a  $D_s D_s^*$  molecule, there is no allowed pion exchange, one can not conclude, from the analysis above, that the  $T_{cc}^s$  should be more deeply bound than the  $T_{cc}$ . On the contrary, if the pion exchange is important for binding the two mesons, the  $T_{cc}^s$  may not be bound.

## 6. Conclusion

In conclusion, we have studied the mass of the  $T_{cc}$  using double ratios of sum rules (DRSR), which are more accurate than the usual simple ratios used in the literature. We found that the molecular currents  $D\bar{D}^* + c.c.$  and  $DD^*$  lead to (almost) the same mass predictions within the accuracy of the method. Since the pion exchange interaction between these mesons is exactly the same, we conclude that if the observed  $X(3872)$  meson is a  $D\bar{D}^* + c.c.$  molecule, then the  $DD^*$  molecule should also exist with approximately the same mass. A recent estimate of the production rate indicates that these states could be seen in LHC experiments [55].

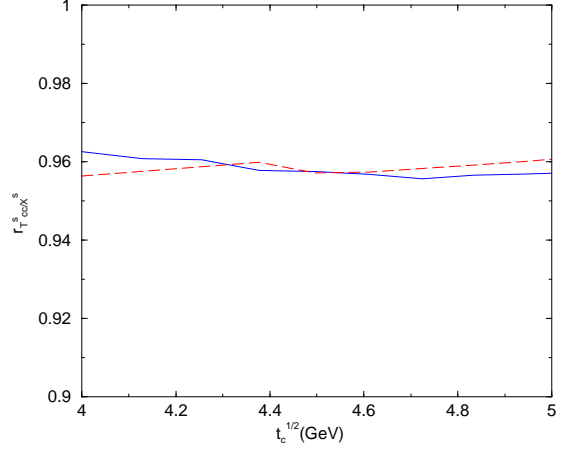


Figure 10: Same as Fig. 2 for  $r_{T_{cc}^s/X^s}$ . The solid and dashed lines are for  $m_c = 1.23$  and  $1.47$  GeV respectively.

We have also studied the double ratio  $r_{T_{bb}/X_b}$  using molecular currents  $\bar{B}\bar{B}^*$  and  $B\bar{B}^* + c.c.$  for  $T_{bb}$  and  $X_b$  respectively. In this case the degeneracy between the two masses is even better than in the charm case. Therefore, we also conclude for the bottom case that if a molecular state  $B\bar{B}^* + c.c.$  exist, then the  $\bar{B}\bar{B}^*$  molecule should also exist with the same mass.

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